

The Triangle Inequality

If ABC is a triangle, then the following inequalities are true:

$$AB \leq BC + AC \qquad BC \leq AB + AC \qquad AC \leq AB + BC$$

What happens to the points when there is an equality ?

Problems

- Problem 1.** Find the perimeter of an isosceles triangle where two sides are 2 and 11.
- Problem 2.** Side AC of triangle ABC has length 3.8, and side AB has length 0.6. If the length of side BC is an integer, what is this length?
- Problem 3.** Prove that the length of any side of a triangle is not more than half its perimeter.
- Problem 4.** It is 6850 miles from Los Angeles to Nanchang, China; 300 miles from Los Angeles to Santa Cruz, CA; 5700 miles from Santa Cruz to Seoul, South Korea; and 850 miles from Seoul to Nanchang, China. How far is it from Los Angeles to Seoul?
- Problem 5.** Find a point inside a convex quadrilateral such that the sum of the distances from the point to the vertices is minimal.
- Problem 6.** Point O is given on the plane of square $ABCD$. Prove that the distance from O to one of the vertices of the square is not greater than the sum of the distances from O to the other three vertices.
- Problem 7.** Prove that the sum of the diagonals of a convex quadrilateral is less than the perimeter but more than half the perimeter.
- Problem 8.** Prove that $|x + y + z| \leq |x| + |y| + |z|$.
- Problem 9.** Prove that $|x + y| + |y + z| \leq |x| + 2|y| + |z|$.
- Problem 10.** Prove that $|x| + |y| + |z| + |w| \geq \frac{1}{2}(|x + y| + |y + z| + |z + w| + |w + x|)$.